THEORY OF EQUILIBRIUM AND STABILITY OF HIGH-CURRENT DISCHARGE IN LOW-CONDUCTIVITY PLASMA

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In this paper we examine the equilibrium and stability of an optically opaque high-current discharge under conditions of strong radiant heat transfer, which ensures uniformity of the temperature in the equilibrium state and rapid temperature equalization during fluctuations. The dispersion equations are obtained and the instability development increments are calculated for the cases of simple cylindrical discharge (z-pinch) and discharge with reverse axial current. The study is made for the limiting case of low-conductivity plasma, which corresponds to discharge parameters which have been proposed for use as plasma light sources [1]. It is shown that maintenance of the discharge for a fairly long time in a state close to equilibrium, when large-scale instabilities capable of altering significantly the equilibrium state have not yet managed to develop, is possible only in a discharge with reverse current. The presence of instabilities of the kink and sausage types in the low-conductivity plasma along with the well-known fact of the existence of such instabilities in high-conductivity plasma [2, 3] suggests that instabilities of this sort are characteristic for plasma of any conductivity.

In connection with the considerable interest in the problem of using high-current pulsed discharges as powerful light sources, a theoretical examination was undertaken in [1, 4] of the equilibrium and stability of discharges in optically dense and optically transparent plasmas with the objective of clarifying their properties at a radiating surface temperature $T \sim (3-10)$ eV and particle density $N \sim 10^{19}$ cm^{-3}. As is known from studies on controlled thermonuclear synthesis, the pinch discharge is hydrodynamically unstable; in this discharge, instabilities of the local overheat, kink, and sausage types develop, which leads to current cutoff and plasma collapse after very short times, usually amounting to a few microseconds (see [3] and the literature cited therein). Therefore, one of the basic problems in using high-current discharges in a plasma as light sources is that of discharge stability.

However, it was noted in [1] that application of the theory presented in [2] to the case in question is not legitimate, since the analysis of [2] applies to a high-temperature, practically infinitely conducting plasma which is transparent for radiation. For the purposes noted above, we are interested in both opaque and transparent plasmas under conditions of rather low temperatures ($T \sim (3-10)$ eV), when finite conductivity effects (diffusion of the electromagnetic fields into the plasma, absence of the current skin-effect, and so on), which can be neglected for high-temperature thermonuclear plasmas, can play a significant role.

1. Problem formulation. Equilibrium and stability of plane and cylindrical discharges in an opaque plasma have been studied on the basis of the equations of single-fluid MHD in [1].

Study of the equilibrium discharge state has shown that the characteristic temperature variation scale x_T (r_T for the cylindrical discharge) differs significantly from the characteristic pressure and density variation scale $x_p(r_p)$. Satisfaction of the inequality $x_T \gg x_p$ ($r_T \gg r_p$) ensures uniformity of the temperature across the discharge section and, consequently, high temperature of the plasma radiating surface. Analysis of fluctuations with wavelengths $\lambda_x \ll x_p$ has shown that the discharge is stable in the geometric-optics approximation. For analysis of fluctuation wavelengths which are comparable with the characteristic dimensions of the system, it is possible to use the approximation in which the temperature fluctuations are practically instantaneously diffused by the high radiant heat conduction. The condition for validity of the approximation is the inequality $x_T^2 c^2 \gg x_p^3 \sigma_0 v_S$.

The stability of both equilibrium and nonequilibrium discharges in cases of high $c^2 < \sigma_0 v_S x_p$ and low $c^2 > \sigma_0 v_S x_p$ conductivity was examined in [1] under the assumption of constant discharge temperature and absence of fluctuations. We note that study of the nonequilibrium-discharge stability is particularly important in high-conductivity plasma, since the establishment of equilibrium with respect to the field, amounting to its equalization across the discharge section, is determined by the skin time $\tau_* \approx 4\pi\sigma_0 x_p^2 c^{-2}$. It is obvious that we can speak of steady-equilibrium stability only if the skin time τ_* is less than the time $\tau \approx x_p/v_S$ characterizing the duration of the discharge formation process, i.e., provided

$$c^2 \gg 4\pi\sigma_0 v_s x_p \tag{1.1}$$

This inequality is not satisfied in high-conductivity plasma. The stability of the low-conductivity plasma, in which the reverse inequality (1.1) holds, can be studied under the assumption of the existence of equilibrium.

In [1] the stability of the equilibrium cylindrical low-conductivity discharge was studied only for the m = 0 mode in the longwave limit. In this paper we present a complete examination of the stability of the cylindrical discharge and of the equilibrium and stability of the discharge with reverse current in a low-conductivity plasma. For the above assumptions concerning high radiant heat conduction, all the processes taking place in such a plasma can be considered isothermal. In this case the system of MHD equations is written in the form

$$\begin{gathered}\frac{\partial\rho}{\partial t} + \operatorname{div}\rho\mathbf{v} = 0, \qquad p = \frac{(1+z)\varkappa}{M}\rho T \equiv v_s^2\rho \\ \rho\left[\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v}\right] = -\nabla p + \frac{1}{4\pi}[\operatorname{rot}\mathbf{B}, \mathbf{B}] \\ \operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \qquad \Delta\mathbf{B} = 0\end{gathered} \tag{1.2}$$

We shall see later that the boundary conditions for (1.2) are obtained directly from (1.2) by using the explicit form of the equilibrium solutions for both the simple cylindrical discharge and the discharge with reverse current.

2. **Simple cylindrical discharge (z-pinch).** As is known, the equilibrium state of the cylindrical discharge is maintained by the current whose self-magnetic field at each point balances the kinetic pressure of the plasma. In the stationary equilibrium state the discharge field E_0 is constant across the discharge section and the hydrodynamic velocity $\mathbf{v}_0 = 0$. The expressions for the equilibrium values of the hydrodynamic quantities have the following form [1]:

$$\begin{gathered}p_0 = p_0(0)\left(1 - \frac{r^2}{r_p^2}\right), \qquad \rho_0 = \rho_0(0)\left(1 - \frac{r^2}{r_p^2}\right) \\ B_0 = \sqrt{4\pi p_0(0)}\,\frac{r}{r_p}, \qquad r_p^2 = \frac{p_0^2(0)\,c^2}{\pi j_0^2}\end{gathered} \tag{2.1}$$

Let us examine the stability of this equilibrium state. Linearizing (1.2) with respect to small deviations of the hydrodynamic quantities from the equilibrium values, which depend on the time and coordinates, in the form $\varphi(r)\exp(-i\omega t + im\varphi + ik_z z)$, we obtain the following system of equations for the perturbations:

$$-i\omega\frac{p_1}{v_s^2} + \frac{1}{r}\frac{\partial}{\partial r}r\rho_0 v_r + \frac{im}{r}\rho_0 v_\varphi + ik_z\rho_0 v_z = 0 \tag{2.2}$$

$$i\omega\rho_0 v_r = \frac{\partial}{\partial r}\left(p_1 + \frac{B_0 B_\varphi}{4\pi}\right) + \frac{B_0 B_\varphi}{2\pi r} - \frac{im}{4\pi r}B_0 B_r \tag{2.3}$$

$$i\omega\rho_0 v_\varphi = \frac{im}{r}p_1 - \frac{B_0 B_r}{2\pi r} \tag{2.4}$$

$$i\omega\rho_0 v_z = ik_z\left(p_1 + \frac{B_0 B_\varphi}{4\pi}\right) - \frac{im}{4\pi r}B_0 B_z \tag{2.5}$$

$$\begin{gathered}\Delta B_r - \frac{B_r}{r^2} - \frac{2im}{r^2}B_\varphi = 0, \qquad \Delta B_\varphi - \frac{B_\varphi}{r^2} + \frac{2im}{r^2}B_r = 0 \\ \Delta B_z = 0, \qquad \operatorname{div}\mathbf{B} = 0\end{gathered} \tag{2.6}$$

Let us examine separately the stability of the modes with m = 0 and m ≠ 0. For m = 0 the equations for B_r and v_φ separate. The solution for B_φ which is finite at zero has the form

$$B_\varphi = C_1 I_1(\beta r), \qquad \beta \equiv |k_z| \tag{2.7}$$

Substituting the expressions for v_r and v_z from (2.3) and (2.4) into (2.2) and using the explicit form of B_φ from (2.7), we obtain for p_1 a second-order nonhomogeneous equation, from which we find the bounded-at-zero solution

$$\begin{gathered}p_1 = C_2 I_0(\alpha r) + \Gamma\frac{\alpha^2}{\alpha^2 - \beta^2}I_0(\beta r) \\ \alpha \equiv \left|\sqrt{k_z^2 - \frac{\omega^2}{v_s^2}}\right| = \left|\sqrt{k_z^2 + \frac{\gamma^2}{v_s^2}}\right| \qquad \left(\Gamma \equiv \frac{C_1 B_0 \beta}{\pi r \alpha^2}\right)\end{gathered} \tag{2.8}$$

Here we have introduced $\gamma = i\omega$ in place of the frequency ω. For unstable solutions $\gamma > 0$.

The problem boundary conditions follow from vanishing of the density ρ_0 at the edge of the plasma. By virtue of the boundedness of the plasma velocity perturbation at the edge, the right-hand sides of (2.3) and (2.5) must vanish for

$r = r_p$, which gives the two boundary conditions

$$\left[\frac{\partial}{\partial r}\left(p_1 + \frac{B_0 B_\varphi}{4\pi}\right) + \frac{B_0 B_\varphi}{2\pi r}\right]_{r=r_p} = 0, \qquad \left[p_1 + \frac{B_0 B_\varphi}{4\pi}\right]_{r=r_p} = 0 \tag{2.9}$$

It is easy to show that the second condition in (2.9) has the obvious physical meaning of conservation of the total current in the discharge during disturbances.

Substituting solutions (2.7) and (2.8) into (2.9), we obtain a system of homogeneous algebraic equations for the coefficients C_1 and C_2, from whose solvability condition we obtain for the m = 0 mode the following dispersion equation:

$$\begin{gathered}(\alpha^2 + \beta^2) I_1(\beta r_p)\ I_0(\alpha r_p) - 2\alpha\beta I_0(\beta r_p) I_1(\alpha r_p) + {}^1/_2 r_p (\beta^2 \\ - \alpha^2)\ [\alpha I_1(\alpha r_p) I_1(\beta r_p) - \beta I_0(\alpha r_p) I_0(\beta r_p)] = 0\end{gathered} \tag{2.10}$$

This equation has a solution in two cases. For the longwave $\beta r_p \ll 1$, $\alpha r_p \ll 1$, where $\alpha \gg \beta$, (2.10) leads to the spectrum

$$\alpha^2 = \pm 2\sqrt{3}\,\frac{\beta}{r_p} \quad \text{or} \quad \gamma^2 = \pm 2\sqrt{3}\,\frac{|k_z|\, v_s^2}{r_p} \tag{2.11}$$

This spectrum coincides with the unstable solution found in [2] for isothermal plasma of infinite conductivity but under the assumption of distributed current. We see from (2.11) that this instability is retained in the low-conductivity plasma.

In the shortwave region $\beta r_p \gg 1$, $\alpha r_p \gg 1$, (2.11) has the unstable root

$$\alpha = \beta + \frac{1}{r_p} \quad \text{or} \quad \gamma^2 \approx \frac{2\,|k_z|\, v_s^2}{r_p} \tag{2.12}$$

The maximum increment of the shortwave fluctuations is limited by the condition of applicability of the radiant-heat-conduction approximation $l < \lambda_z$; therefore $\gamma_{max} < 2v_s^2/lr_p$, where l is the Rosseland quantum mean free path.

All the higher modes of the longwave instabilities, and also the shortwave modes of the type $v^2 \sim k_z^2 v_s^2$ which exist in high-conductivity plasma, are stabilized in the low-conductivity plasma. We note that the nature of the spatial behavior of the solutions (2.7), (2.8) for the disturbances differs from the case of the high conductivity. They decay monotonically from the edge into the depth of the plasma, while in the case of the high conductivity there was an oscillatory variation of the solutions in the body of the plasma. A consequence of this is the effective increase of the time for development of shortwave instabilities in the depth of the plasma. As for the rapidly developing small-scale surface disturbances, they are not dangerous. For the reasons mentioned, we can consider that the cylindrical low-conductivity discharge is more stable with respect to axisymmetric instabilities than the high-conductivity discharge.

Now let us investigate the $m \neq 0$ modes, which are kink and spiral perturbations of the pinch discharge. Solving (2.2)–(2.6), we obtain

$$\begin{gathered}B_z = C_1 I_m(\beta r), \qquad B_r = \frac{C_2}{\beta r} I_m(\beta r) - iC_1 \operatorname{sign} k_z \frac{\partial I_m(\beta r)}{\partial \beta r} \\ p_1 = C_3 I_m(\alpha r) + \Gamma \frac{\alpha^2}{\alpha^2 - \beta^2} I_m(\beta r), \qquad \Gamma \equiv \frac{i\beta B_0}{\pi m \alpha^2 r} C_2\end{gathered} \tag{2.13}$$

These solutions of (2.2)–(2.6) differ from the general solutions in the requirement that the perturbations be finite for r = 0. The boundary conditions, as for the case m = 0, are obtained directly from (2.3)–(2.5) and have the form

$$\begin{gathered}\left[\frac{\partial}{\partial r}\left(p_1 + \frac{B_0 B_\varphi}{4\pi}\right) + \frac{B_0 B_\varphi}{2\pi r} - \frac{im}{4\pi r} B_0 B_r\right]_{r=r_p} = 0 \\ \left[\frac{im}{r} p_1 - \frac{B_0 B_r}{2\pi r}\right]_{r=r_p} = 0, \quad \left[p_1 + \frac{B_0 B_\varphi}{4\pi} - \frac{m}{4\pi r k_z} B_0 B_z\right]_{r=r_p} = 0\end{gathered} \tag{2.14}$$

The condition of solvability of (2.14) relative to the coefficients C_1, C_2, and C_3 leads to the following dispersion equation for determining the plasma fluctuation spectra:

$$\alpha I_{m+1}(\alpha r_p) \left[\frac{\beta^2}{r_p^2} I_m(\beta r_p) \frac{\partial I_m(\beta r_p)}{\partial r_p} + \frac{\alpha^2 - \beta^2}{4 r_p} \left(\frac{\partial I_m(\beta r_p)}{\partial r_p}\right)^2\right] +$$

$$+ I_m(\alpha r_p)\Big\{I_m(\beta r_p)\frac{\partial I_m(\beta r_p)}{\partial r_p}\Big[\frac{m\beta^2}{r_p^3} - \frac{(\alpha^2-\beta^2)\beta^2}{4r_p} - \frac{m^2(\alpha^2-\beta^2)}{4r_p^3}\Big]$$
$$+\left(\frac{\partial I_m(\beta r_p)}{\partial r_p}\right)^2\Big[\frac{(\alpha^2-\beta^2)m - 2(\alpha^2+\beta^2)}{4r_p}\Big] + \frac{(\alpha^2-\beta^2)m^2}{2r_p^4}I_m^2(\beta r_p)\Big\} = 0 \tag{2.15}$$

In the region of most dangerous longwave oscillations $\beta r_p \ll 1$, $\alpha r_p \ll 1$, expanding the terms of (2.15) into a series in powers of βr_p and αr_p up to and including terms of fourth order, we obtain the unstable roots

$$\alpha^2 = \beta^2\frac{m+2}{m} \quad \text{or} \quad \gamma^2 = \frac{2}{m}k_z^2 v_s^2 \tag{2.16}$$

We note that this solution is not valid for the m = 0 mode, analyzed above. In the shortwave limit $|k_z| r_p \gg 1$ the unstable oscillation spectrum has the form

$$\alpha^2 = \beta^2 + \frac{2\beta}{r_p} \quad \text{or} \quad \gamma^2 = \frac{2|k_z|v_s^2}{r_p} \tag{2.17}$$

This spectrum coincides with the corresponding spectrum for the m = 0 mode (see (2.12)), i.e., the shortwave instabilities have the same increment for all the azimuthal modes. It follows from (2.16) and (2.17) that in the low-conductivity plasma excitation of fluctuations with any m is possible, while in the high-conductivity plasma in the absence of an external magnetic field the modes with $m \geq 2$ are not excited in the longwave limit, being stabilized by the current self-magnetic field [5]. In this case the discharge in the low-conductivity plasma is less stable with respect to disturbances with $m \neq 0$ than is the discharge in the ideally conducting plasma.

3. Discharge with reverse current. Let us examine the discharge with reverse current in an optically dense plasma under conditions of radiant heat conduction. Such a system consists of a coaxial cylindrical layer carrying the current and a massive metal conductor located along the axis of the system, through which current flows in the reverse direction.

It will be shown later that such a discharge has considerably greater stability than the simple cylindrical (z-pinch) discharge. This fact was known from the very first few studies of discharges with reverse current in a high-temperature, ideally conductive plasma [6]. Moreover, the possibility of creating in such systems a very large radiating surface makes them more promising for use as powerful light sources than the z-pinch.

The system of equations describing the equilibrium discharge with reverse current has the same form as for the z-pinch [1]. However, in solving this system we must consider the reverse current azimuthal field.

Considering the plasma temperature constant as a result of the high radiant heat conduction, we find the following distribution of the equilibrium quantities in the discharge with reverse current:

$$\begin{aligned} B_0 &= \frac{2\pi}{c}\sigma_0 E_0 r\left(1 - \frac{R_0^2}{r^2}\right) \\ p_0 &= p_m + \frac{\pi R_0^2\sigma_0^2 E_0^2}{c^2}\left(1 - \frac{r^2}{R_0^2} + \ln\frac{r^2}{R_0^2}\right) \\ \rho_0 &= \rho_m + \frac{\pi R_0^2\sigma_0^2 E_0^2}{c^2 v_s^2}\left(1 - \frac{r^2}{R_0^2} + \ln\frac{r^2}{R_0^2}\right) \end{aligned} \tag{3.1}$$

Here p_m and ρ_m are the maximum values of the plasma pressure and density, which are reached at the point $r = R_0$. We see from (3.1) that the discharge is concentrated in the region $r_1 \leq r \leq r_2$, where r_1 and r_2 are the roots of the equation $p_0(r) = 0$. The quantities R_0 and r_1 are connected by the relation

$$R_0^2 = r_1^2 + \frac{I_0}{\pi j_0} \tag{3.2}$$

where j_0 is the discharge current density and I_0 is the total axial current. In the limiting case $R_0 \to 0$ (i.e., $I_0 \to 0$ and $r_1 \to 0$ as well), formulas (3.1) become (2.1) for the simple z-pinch.

For large I_0 the equilibrium radius $R_0 \gg r_2 - r_1$. In this case it is convenient to introduce the variable $x = r - R_0$ ($|x| \ll R_0$). Expanding (3.1) into a series in powers of x/R_0, we obtain

$$B_0 = \frac{4\pi}{c}\sigma_0 E_0 x = \sqrt{8\pi p_m}\,\frac{x}{x_p}$$

$$p_0 = p_m\left(1 - \frac{x^2}{x_p^2}\right), \quad \rho_0 = \rho_m\left(1 - \frac{x^2}{x_p^2}\right), \quad x_p^2 = \frac{p_m c^2}{2\pi\sigma_0^2 E_0^2} \tag{3.3}$$

These formulas coincide exactly with the distribution of the equilibrium quantities in the plane surface discharge [1]. Equivalence of the inverse pinch for high axial currents to the plane discharge holds provided

$$\frac{R_0}{x_p} = \frac{I_0 j_0}{p_m c^2} = \frac{I_0 \sigma_0 E_0}{p_m c^2} \gg 1 \tag{3.4}$$

In this case the equilibrium radius R_0 equals

$$R_0 = \frac{I_0}{\sqrt{2\pi p_m} c} \tag{3.5}$$

and the following relation is satisfied:

$$I_n = 4\pi R_0 x_p j_0 = 2I_0$$

Now let us express the quantities T_m and ρ_m (or N_m) in terms of the total number of particles trapped in the discharge and the total discharge current. We have for the total number of particles per centimeter length of the plasma layer

$$N_n = 4\pi R_0 \int_0^{x_p} dx N_m\left(1 - \frac{x^2}{x_p^2}\right) = \frac{8\pi}{3} R_0 x_p N_m \tag{3.6}$$

We find the plasma temperature using the energy balance between radiation from the plasma surface and ohmic heating,

$$\hat{\sigma} T_m^4 = \sigma_0 x_p E_0^2 \tag{3.7}$$

Using (3.6) and (3.7), we can show that the discharge current in the inverse pinch has no upper limit, as does the z-pinch in the high-temperature ideally conducting plasma [7]. The plasma temperature is connected with the total number of trapped particles and the total discharge current by the relation

$$T_m = \frac{I_n^3}{12 I_0} \frac{1}{(1+z)\varkappa c^2 N_n} \frac{x_p}{R_0} \tag{3.8}$$

Just as in the simple z-pinch, in the optically dense plasma in the reverse pinch increase of the discharge current leads to increase of the plasma temperature and thereby increase of the radiation power.

Finally, we note that the condition for weak nonuniformity of the temperature $x_T^2 \gg x_p^2$ and the condition for applicability of radiant heat conduction $x_p > l_0$ in the case in question ($R_0 \gg x_p$) coincide with the corresponding conditions for the flat pinch [1].

Analysis of the reverse current discharge stability is based on the same system of equations (2.2)–(2.6) as for the cylindrical discharge, with the difference that the boundary conditions (2.9) and (2.14) are now posed at both boundaries of the discharge, at the points r_1 and r_2 (or $x = \pm x_p$ for the case $R_0 \gg x_p$). The solution of the posed problem involves tedious mathematical computations. Final results can be obtained only for the modes with $m = 0$. In this case the dispersion equation can be written in the form of vanishing of the determinant

$$\begin{vmatrix} u_1 & v_1 & \chi_1 & \xi_1 \\ u_2 & v_2 & \chi_2 & \xi_2 \\ p_1 & t_1 & k_1 & s_1 \\ p_2 & t_2 & k_2 & s_2 \end{vmatrix} = 0 \tag{3.9}$$

where $\varphi_1 \equiv \varphi(r_1)$, $\varphi_2 \equiv \varphi(r_2)$ and the functions themselves in the determinant are

$$u = \alpha I_1(\alpha r), \qquad \chi = \frac{j_0}{c} \frac{\alpha^2 + \beta^2}{\alpha^2 - \beta^2} I_1(\beta r) + \frac{\beta B_0}{4\pi} I_0(\beta r)$$

$$v = -\alpha K_1(\alpha r), \qquad \xi = \frac{j_0}{c} \frac{\alpha^2 + \beta^2}{\alpha^2 - \beta^2} K_1(\beta r) - \frac{\beta B_0}{4\pi} K_0(\beta r)$$

$$p = I_0(\alpha r), \qquad k = \frac{2j_0\beta}{c(\alpha^2 - \beta^2)} I_0(\beta r) + \frac{B_0}{4\pi} I_1(\beta r)$$

$$t = K_0(\alpha r), \qquad s = -\frac{2j_0\beta}{c(\alpha^2 - \beta^2)} K_0(\beta r) + \frac{B_0}{4\pi} K_1(\beta r) \tag{3.10}$$

Here I_ν is the Bessel function of imaginary argument, K_ν is the Macdonald function, α and β are introduced following (2.8).

For the shortest wavelength case, when $\beta(r_2 - r_1) \gg 1$, the dispersion equation (3.9) reduces to the form

$$(\alpha^2 + \beta^2) I_0(\alpha r_2) I_1(\beta r_2) - 2\alpha\beta I_1(\alpha r_2) I_0(\beta r_2) + \left(1 - \frac{R_0^2}{r_2^2}\right) \times (\beta^2 - \alpha^2)\frac{r_2}{2}\{\alpha I_1(\alpha r_2) I_1(\beta r_2) - \beta I_0(\alpha r_2) I_0(\beta r_2)\} = 0 \tag{3.11}$$

In the sought region, assuming r_2 close to R_0, $r_2 - R_0 = x_p \ll R_0$, we find the unstable root

$$\alpha = \beta + \frac{1}{R_0} \quad \text{or} \quad \gamma^2 \approx \frac{2|k_z| v_s^2}{R_0} \tag{3.12}$$

The maximum increment of the shortwave oscillations is

$$\gamma_{\max} \approx 2v_s^2 / R_0 l$$

where l is the Rosseland quantum mean free path. In the region $\beta R_0 \gg 1$, $\beta x_p \ll 1$ (assuming that $R_0 \gg x_p$) Eq. (3.9) has no solution. Finally, for longwave oscillations, when $\beta R_0 \ll 1$, we find from (3.9)

$$\gamma^2 = \frac{2|k_z| v_s^2}{R_0} < \frac{v_s^2}{R_0^2} \tag{3.13}$$

Thus the shortwave instabilities with wavelength less than the discharge-layer thickness have the largest increment. However, as we noted previously, in examining the z-pinch such instabilities do not present any great danger for the discharge as a whole. The dangerous longwave instabilities with $c|k_z|R_0 \ll 1$ develop considerably more slowly than in the cylindrical pinch (for sufficiently large R_0).

Analysis of the $m \neq 0$ modes is very complicated. However, in the shortwave limit, when $|k_z|x_p \gg 1$, it is possible to show that the increment is independent of the azimuthal number m and has the same form as in the z-pinch with the replacement $r \to R_0$, i.e.,

$$\gamma^2 = \frac{2|k_z| v_s^2}{R_0} \tag{3.14}$$

According to the above theory, it is possible to realize an equilibrium discharge in a low-conductivity plasma provided the time for development of the instabilities is longer than the time for penetration of the discharge field into the plasma, i.e.,

$$\gamma 4\pi\sigma_0 a_1^2 < c^2 \tag{3.15}$$

For the cylindrical discharge $a_1 = r_p$ $y \sim v_s/r_p$; for the inverse pinch $a_1 \approx x_p$, $y \sim v_s/R_0$.

The duration of the existence of the equilibrium state is determined by the instability development times. In the simple z-pinch at temperatures $T \approx (3-10)$ eV and radius $r_p \approx 2-3$ cm the instability development time $\tau_1 \approx 10^{-5}$ sec = 10 μsec. Large values of r_p are not achievable because of violation of condition (3.15). As for the inverse pinch, it can be maintained for a fairly long time $\tau_2 \sim R_0/v_s \approx 50-100$ μsec as a result of the large value of R_0. In combination with the large radiating surface this makes it possible to consider the reverse-current discharge as a possible plasma light source.

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